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Convective heat transfer in shallow, gas-filled porous media: experimental investigation

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Abstract—Measurements of Nusselt numbers in shallow, air-filled porous beds bounded by horizontal isothermal surfaces are reported. The Rayleigh number range covered was from 5 to 150 and the number of layers of spheres investigated was from 1 to 5. Critical Rayleigh numbers were found to be close to $4\pi^2$ except in the case of the one-layer bed. The ratio of particle diameter to bed height had a strong influence on the Nusselt number.

1. INTRODUCTION

A NUMBER of authors report extensive measurements on natural convection in liquid-filled packed beds, refs. [1–4]. However, no useful data have been found for gas-filled beds, the experiments of Schneider [4] being below the critical Rayleigh number, and the data of Katto and Matsuoka [5] applying only near the critical Rayleigh number.

The experimental equipment used have usually been a packed bed of spheres contained between horizontal isothermal plates, although a small amount of data exists for other configurations. Generally the horizontal dimensions of the container have been large (several times) compared with the height. Data are correlated by a function of the form

$$Nu^* = f(Ra^*),$$

the additional parameter H/L being neglected as the beds are considered to be essentially infinite in extent. The parameters Ra^* and H/L arise from a model of the system where the solid and adjacent fluid temperatures are assumed equal.

A model proposed by Combarnous [6] introduces parameters additional to Ra^* and H/L, which include the solid and fluid conductivities and the local fluid-to solid convective heat transfer coefficient. Chan and Banerjee [7] obtained good agreement between some data of Combarnous [2] and their numerical model, by employing a relation of the form

$$j \equiv \frac{Nu}{Re_p \cdot Pr} \cdot Pr^{2/3} = a\, Re_p^b$$

to establish the local heat transfer coefficient.

From the models of Combarnous [6] and Chan and Banerjee [7] and the spread of data from liquid-filled beds, it is clear that the simple model yielding $Nu^* = f(Ra^*)$ is inadequate and that properties such as the fluid Prandtl number and the fluid and solid conductivities affect bed behaviour in ways not described by Ra^*. Since the effective conductivity of a packed bed k results from a complex interaction of these factors, it is not possible to infer the behaviour of gas-filled packed beds from data derived from liquid-filled packed beds. An additional factor preventing such inferences is that radiation heat transfer is present in gas-filled packed beds, but absent in the liquid-filled experimental beds.

This study was initially concerned with the gathering of data on gas-filled porous media which, it is hoped, will lead to an improved model for flow within these beds.

Interest in shallow beds arises from work on energy storage systems, either under-floor air-filled gravel beds [14] which heat the building by transfer up through the floor, or high-temperature energy storages which receive heat at the bottom of the container and reject heat at the top [15]. In both these systems, transfer within the porous medium is primarily by natural convection and certainly in the case of the under-floor gravel bed, and possibly in the case of the high-temperature storage, the bed height may be as low as three particle diameters.

2. DESCRIPTION OF APPARATUS

The apparatus used comprises two isothermal plates 450×450 mm, with a section 125×125 mm in the heated plate having accurately measured power supplied to it. Two temperature-controlled baths supply water to tubes in the isothermal plates. The small inset plate is mounted with a flux meter between it and the large plate. When the heat flux meter is nulled, then electric power supplied to the small plate is all transferred to the porous medium and ultimately to the other isothermal plate. This apparatus is placed in a pressure vessel which can vary the gas pressure to between 10 and 700 kPa. A micro-computer both controls the experiment and logs and processes the data.

A more complete description of the system and its experimental uncertainties is given by Peck [8]. Figure 1 shows the apparatus.

The uncertainty in Nu^* is $\pm 2\%$ resulting from

NOMENCLATURE

- a constant
- b constant
- C_k Kozeney–Carman constant
- C_p specific heat of fluid [J kg^{-1} K^{-1}]
- d_h hydraulic diameter of pores in porous medium [m]
- g gravitational acceleration [m s^{-2}]
- H spacing between isothermal plates [m]
- j Colburn j-factor
- k effective conductivity of packed bed [W m^{-1} K^{-1}]
- k_f conductivity of fluid [W m^{-1} K^{-1}]
- k^* stagnant conductivity of packed bed [W m^{-1} K^{-1}]
- L width of convective cell [m]
- Nu^* k/k^*
- O_z total surface area within packed bed including enclosure and isothermal surfaces [m^2]
- Pr fluid Prandtl number
- Ra^* Rayleigh number, $g\beta \dfrac{\rho^2 C_p}{\mu} \dfrac{\kappa}{k^*} \Delta T\, H$
- Re_p Reynolds number based on sphere diameter
- V_z volume of voids in packed bed [m^3]

Greek symbols

- β coefficient of volumetric expansion of fluid [K^{-1}]
- ΔT temperature difference between isothermal surfaces [K]
- ε void fraction of packed bed
- κ permeability of packed bed [m^2]
- ρ density of fluid [kg m^{-3}]
- μ viscosity of fluid [N s m^{-2}]

uncertainties in k^* and k, traceable to uncertainty in the measured temperatures and electrical power. The uncertainty in Ra^* is $\pm 10\%$ due to uncertainties in temperature and pressure measurements and in the geometric description of the bed, particularly void fraction and particle diameter.

Since $Ra^* \propto \rho^2$, varying the pressure from 10 to 700 kPa allows a range of 5 to 150 in Ra^* to be covered using spheres of about 18 mm in diameter. This is much more convenient than changing temperature or spacing to achieve the same Ra^* range, and since the plate temperatures remain nearly constant, long heat up and cool down times are avoided.

The heat flow measured by the small inset plate may

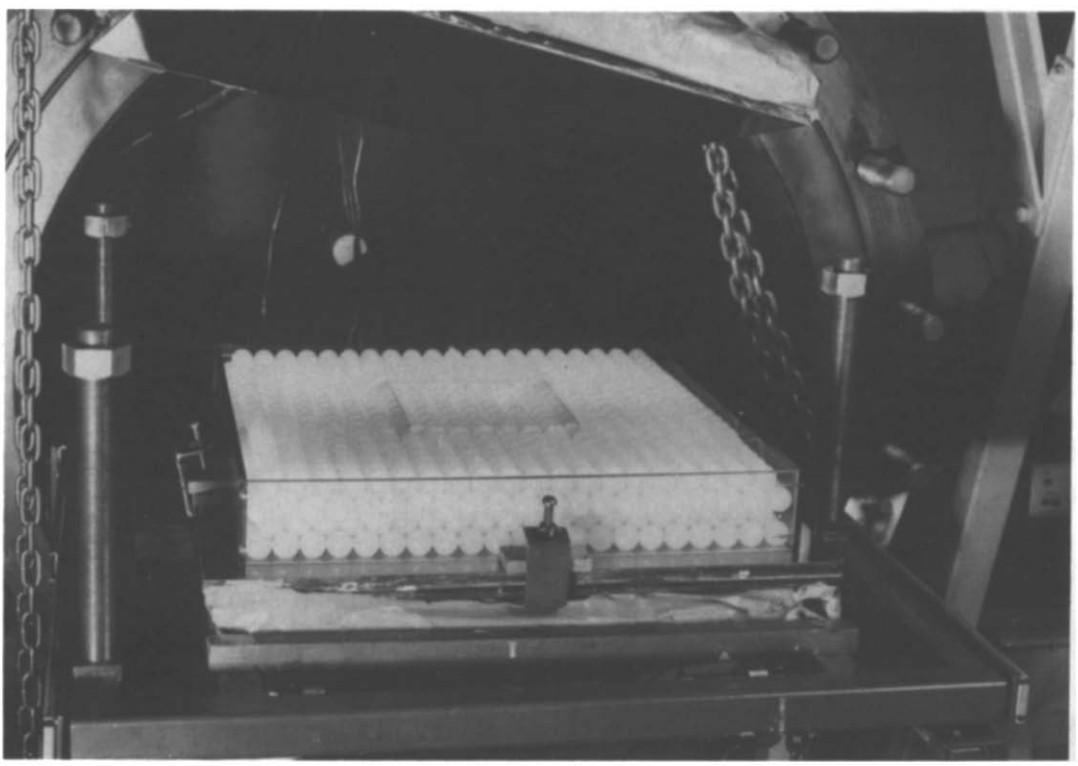

FIG. 1. Photograph of open pressure vessel with sphere array mounted on heated isothermal plate and inner enclosure in position. The cooled isothermal plate is above the array.

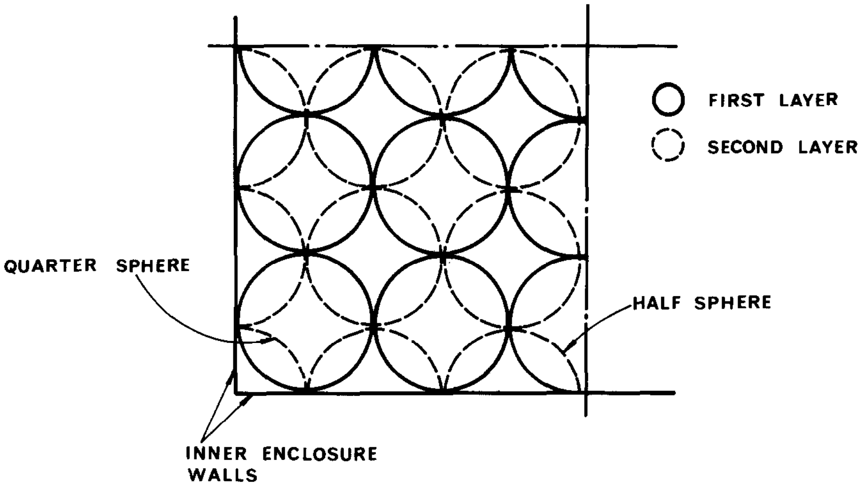

FIG. 2. Diagram showing array geometry as viewed from above.

not represent an average over the whole plate. For the packed bed experiment square enclosures of polycarbonate were constructed, one 130 x 130 mm and the other 390 x 390 mm. The smaller was just a little larger than the small inset plate and used polycarbonate walls 1 mm thick. When in place it ensures that the inset plate measured the heat flow from discrete cells only. The larger covered most of the isothermal plates and had walls 5 mm thick. For one set of experiments spheres were placed in the smaller enclosure, and in the space between the smaller and larger enclosure. For the other set, the smaller enclosure was removed. Figure 1 shows the experimental bed with small and large enclosures in place.

Using the small enclosure, then discrete cells are contained within it and so the power to the small heated plate must be transferred by the discrete cells. Without the enclosure in place, then parts only of convection cells may be in contact with the small plate. In this situation it is not certain that the power to the plate represents the average heat transfer for discrete cells.

Between one and five layers of spheres were used in the experiments, the maximum number being determined by the maximum isothermal plate spacing and diameter of suitable spheres. The array geometry used is shown in Fig. 2, with half and quarter spheres cut to fit into sides and vertical edges. This eliminated the variation in void fraction normally encountered near the walls and corners of vessels containing packed beds. This use of part spheres was not done for the isothermal surfaces.

To ensure that a large Rayleigh number can be obtained, the sphere material needs to be of low conductivity. This is shown by the model of stagnant conductivities of gas- and liquid-filled packed beds due to Yagi *et al.* [9]. The only suitable material found was polypropylene, the sphere diameter being about 18.4 mm.

The same spheres and packing geometry were used throughout to produce adiabatic walls on the inner enclosure.

Before conducting the packed bed experiments, a check was made on the critical Rayleigh number for a horizontal air layer. The established value, [10, 12], is 1708 and the value measured in the apparatus was 3% higher than this. This is within the $\pm 5\%$ error band for this experiment.

3. EXPERIMENTAL METHOD

For a given number of sphere layers, the heights of the polycarbonate walls were cut to ensure that the top layer of spheres was just touching the upper isothermal plate. When the bottom layer of spheres was packed the spacings between the side walls of the enclosure were adjusted to give a tight packing. Knowing the dimensions of the enclosures, the isothermal plate spacing and the size of the particles, the void fraction could be determined. The relevant physical dimensions are shown in Table 1.

For all experiments, the temperature difference between the isothermal plates was about 30°C with a mean temperature of about 45°C. To vary the Rayleigh number the air pressure inside the pressure vessel was varied. Once the pressure was stabilized, steady state was determined from the output of the heat flux meter. Providing it varied between preset bounds with electric power to the heated plate above it held constant, then steady state was assumed. Isothermal plate temperatures, heat flux meter voltage, air pressure and power to the heater plate were then monitored over a 1-h period to determine average conditions.

For a given bed geometry, that is with a given number of layers and with the inner enclosure either in place or removed, the stagnant conductivity k^* was determined by measuring the bed conductivity at pressures well below the estimated pressure corresponding to the critical Rayleigh number of $4\pi^2$ (see for example Katto

Table 1. Dimensions of test beds (sphere diameter 18.4 mm)

No. of layers	Height (mm)	Inner enclosure (mm × mm)	Outer enclosure (mm × mm)	Void fraction
1	19.3	129.4 × 129.4	389.7 × 389.7	0.503
2	32.4	129.3 × 129.3		0.409
3	45.4	129.5 × 129.5		0.369
4	58.2	129.3 × 129.3		0.341
5	71.3	129.6 × 129.6	389.4 × 389.7	0.332

and Matsuoka [5]). As will be shown later, Rayleigh numbers much lower again were required for the one-layer bed.

These values of k^* agreed within the uncertainty bands of 1.5% and were averaged to provide the value of k^* used in determining the actual Ra^* and Nu^* values. Then the pressure was increased to give a satisfactory number of test points, particularly at Rayleigh numbers near the critical value. This allowed the critical Rayleigh number to be determined.

Values of Ra^* and Nu^* were measured for 1–5 layers of spheres and with the inner and outer enclosures in place. The experiments were repeated for the one- and five-layer beds with the inner enclosure removed.

4. ANALYSIS OF RESULTS

The values of the stagnant conductivities are shown plotted in Fig. 3 against the ratio H/d_p. This does not correspond to the number of layers due to the packing used. The most probable error in k^* is $\pm 1.5\%$. Calculated values using the method of Yagi *et al.* [9] are also shown. For these calculations the thermal conductivity of polypropylene was taken as 0.12 W m^{-1} K^{-1} from [11], and emittance of 0.9 and 1 used since data on this property at the appropriate wavelengths could not be found.

As can be seen the discrepancy is about 10% for five layers using an emittance of 0.9, which is acceptable for a general correlation of the type used. Even better agreement was obtained using an emittance of 1. The low values measured particularly for one- and two-layer beds are assumed to be due to the influence of the highly reflective isothermal plates which have an emittance of about 0.05. The shape of the measured curve is consistent with a decline in influence of these plates as the number of layers increases.

Additional support for this contention is derived

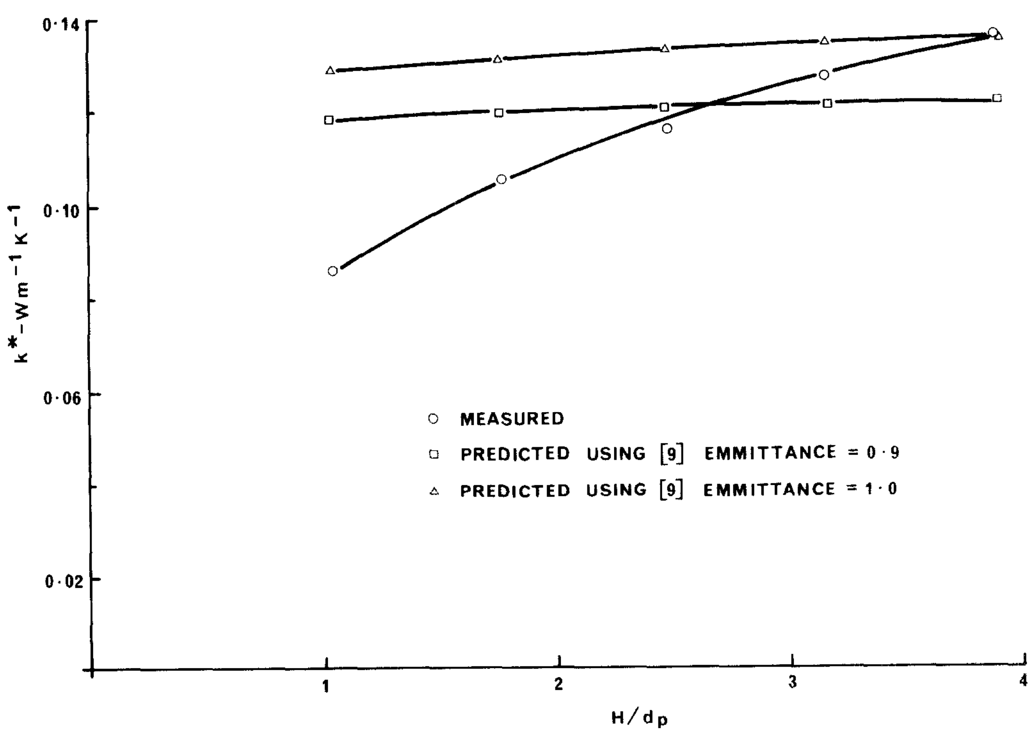

FIG. 3. Graph showing measured and predicted stagnant conductivities as functions of bed height-to-particle diameter ratio.

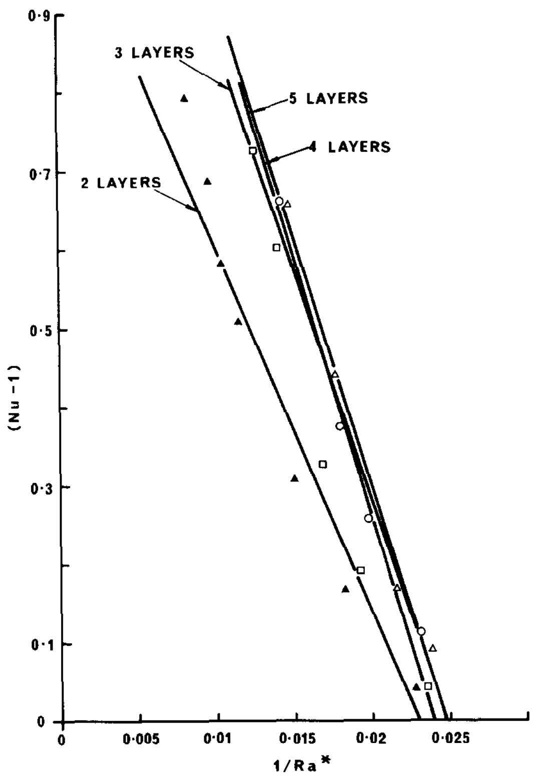

FIG. 4 (a). Data points and regression lines to determine critical Rayleigh numbers for two–five-layer beds. Small enclosure in place.

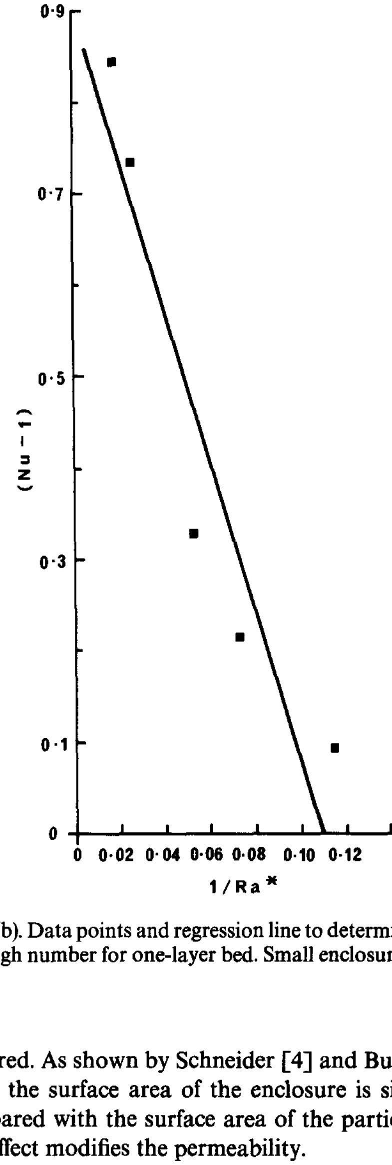

FIG. 4(b). Data points and regression line to determine critical Rayleigh number for one-layer bed. Small enclosure in place.

from the calculated radiation heat transfer directly from plate to plate in the one-layer case. As a simple approximation the sphere layer is replaced by a perforated plate so that the open area is $(1-\pi/4)$ of the total. For black bounding plates with temperatures of 30° and 60°C the radiation heat transfer is about 47 W m^{-2} translating to an effective increase in measured conductivity of the bed of 3×10^{-2} W m^{-1} K^{-1}. This can be compared with the difference between measured and predicted (emittance = 1) values of k^* for one layer of about 5×10^{-2} W m^{-1} K^{-1}. In the analysis, radiation between bounding plates and spheres, which will increase the net radiation heat transfer, has been neglected. This simple calculation serves to show that the use of high reflectance bounding plates reduces heat transfer rates in the order of the discrepancies between measured and predicted values of k^* for one layer of spheres, and is a plausible explanation for this effect.

In calculating the values of Ra^*, the permeability κ is required. As shown by Schneider [4] and Buretta [3], when the surface area of the enclosure is significant compared with the surface area of the particles, then this effect modifies the permeability.

Then,

$$\kappa = \frac{1}{16}\,\varepsilon\,\frac{d_h^2}{C_k}$$

where

$$d_h = 4\,\frac{V_z}{O_z}.$$

These relationships were employed together with a value of C_k of 4.94 as used by Schneider and Buretta.

To estimate the critical Rayleigh number, a least-squares fit to Nu^*-1 as a function of $1/Ra^*$ was made. This follows the functional relationship between Nu^* and Ra^* near the critical value, as determined using the power integral method [12]. The data points and regression lines are shown in Fig. 4 and the values of the critical Rayleigh number given in Table 2. These values depart from the theoretical value of $4\pi^2$ by less than the most probable error in Ra^* of $\pm 10\%$, except in the case of the one-layer bed. This uncertainty is larger than the

Table 2. Critical Rayleigh numbers

No. of layers	Critical Rayleigh number
1	10
2	43
3	40
4	42
5	40

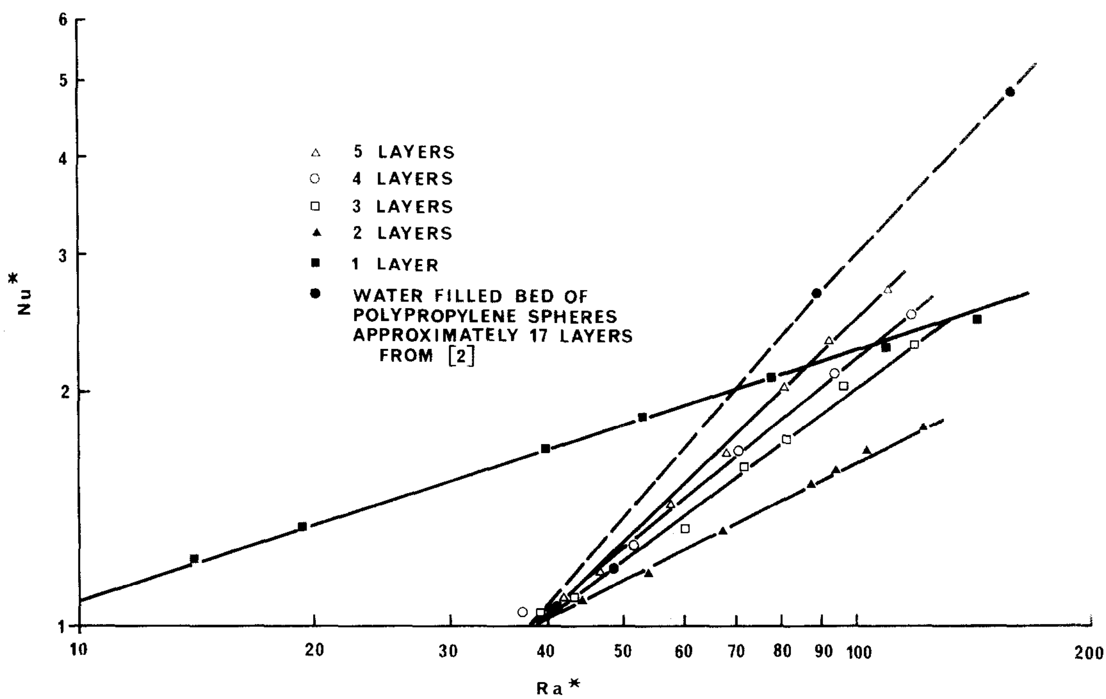

FIG. 5. Graph showing Nu^* as a function of Ra^* for one–five-layer beds in small enclosure.

$\pm 5\%$ for the horizontal air layer, due as noted previously to the additional uncertainties relating to the geometry of the packing, for example in the particle diameter and void fraction both of which have a marked effect on the permeability.

Plots of Nu^* vs Ra^* are shown in Fig. 5. As noted above, the most probable error in Nu^* is $\pm 2\%$ and the most probable error in Ra^* $\pm 10\%$.

5. DISCUSSIONS OF RESULTS AND CONCLUSIONS

With the exception of the one-layer case, the experiment confirms $4\pi^2$ as the critical Rayleigh number for these shallow beds. The discrepancy at one layer was noted by Katto and Matsuoka [5]. Clearly, a one-layer bed does not resemble a continuum, and it is of interest to compare it with an air gap containing a honeycomb used for convection suppression. Using a Rayleigh number defined as,

$$Ra = g\beta \frac{\rho^2 C_p}{\mu} \frac{H^3}{k_f} \Delta T,$$

then the critical value for the one-layer case is 28,000. Data from Hollands *et al.* [13] shows that a honeycomb of aspect ratio (height to wall spacing) of 2 : 1 yields a critical Rayleigh number about the same. The total surface area (including isothermal boundaries) of a hexagonal honeycomb with height 36 mm is about the same as for the single-layer packed bed with sphere diameters 18.4 mm. This shows that concepts based on continuum models of packed beds are inappropriate for a single layer, and it is more reasonable to view such a structure as belonging to that class of geometries including honeycombs.

Since the predicted value of $4\pi^2$ is derived from a continuum model, it appears that viewing packed beds as shallow and two layers as continua at Rayleigh numbers near critical is a valid approach.

The data for 2–5 layers shows a consistent trend of increasing Nu^* with number of layers, at a given value of Ra^*. Work in progress suggests that this effect can be explained by the role of the inertial terms in the momentum equation, an effect due to the relatively small surface area per unit volume in these beds. That is, the viscous shear is not dominant, as would be the case in beds with a larger number of rows, having the same range of Ra^*.

A comparison between measured values of Nu^* for one- and five-layer beds with the inner enclosure in place and removed, is shown in Fig. 6. For the five-layer case with the inner enclosure removed, the data was not repeatable above Ra^* of about 100. The reason for this is not known. Below this value, and for the one-layer case, the data follows the same trend as that when the inner enclosure was in place, although the values of Nu^* at a given Ra^* are lower by about 0.15 to 0.2. This may be due to the convective cells overlapping the small inset plate used for heat flow measurement. It is reasonable to conclude that the effects observed for the confined beds apply to unconfined ones.

Data from Combarnous [2] for a water-saturated bed of polypropylene spheres, are shown in Fig. 5. These experiments were made using 4-mm diameter spheres in a bed 53.5 mm high. It can be seen that all data from the air-saturated beds lie below those for the

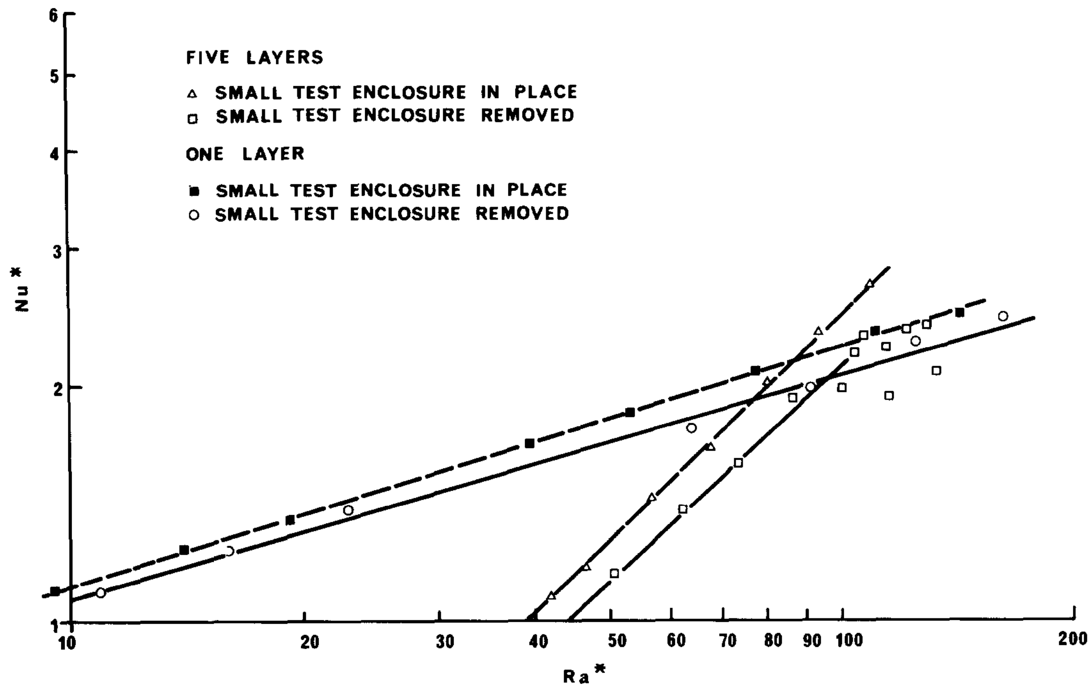

FIG. 6. Graph showing Nu^* as a function of Ra^* for one- and five-layer beds and with inner enclosure in place and removed.

water-saturated bed, although as the number of layers increases the air-saturated data tends to the water-saturated data. An air-saturated bed of 17 layers or so corresponding to Combarnous' data, may be very similar in behaviour to a water-saturated bed, at least for values of Ra^* up to a few hundred.

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CONVECTION THERMIQUE DANS DES MILIEUX POREUX PEU PROFONDS ET EMPLIS DE GAZ. ETUDE EXPERIMENTALE

Résumé—On mesure les nombres de Nusselt dans des lits poreux, remplis de gaz, peu profonds, limités par des surfaces horizontales et isothermes. Le nombre de Rayleigh varie entre 5 et 150 et le nombre de couches de sphères entre 1 et 5. Les nombres de Rayleigh critiques sont proches de $4\pi^2$, excepté dans le cas d'une seule couche. Le rapport du diamètre de particule à la hauteur du lit a une forte influence sur le nombre de Nusselt.

KONVEKTIVER WÄRMEÜBERGANG IN FLACHEN GASGEFÜLLTEN MEDIEN. EXPERIMENTELLE UNTERSUCHUNG

Zusammenfassung—Es werden Messungen der Nusselt-Zahl in flachen luftgefüllten porösen Schichten, die von horizontalen isothermen Oberflächen begrenzt sind, vorgestellt. Die Rayleigh-Zahl wurde in einem Bereich von 5 bis 150 und die Zahl der Kugelschichten von 1 bis 5 variiert. Die kritische Rayleigh-Zahl beträgt ungefähr 4 π^2, ausgenommen für den Fall der Einlagenschicht. Das Verhältnis von Partikeldurchmesser zu Schichthöhe hat einen starken Einfluß auf die Nusselt-Zahl.

КОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС В ТОНКИХ, ЗАПОЛНЕННЫХ ГАЗОМ ПОРИСТЫХ СРЕДАХ. ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ

Аннотация—Приведены результаты измерения чисел Нуссельта для тонких, заполненных воздухом пористых слоев, ограниченных горизонтальными изотермическими поверхностями. Исследуемый диапазон чисел Рэлея составлял 5–150, исследуемая пористая среда состояла из различного количества слоев (от 1 до 5) сферических частиц. Найдено, что критические значения числа Рэлея равнялись примерно $4\pi^2$, за исключением среды, состоящей из одного слоя. Отношение диаметра частиц к высоте слоя оказывалос сильное влияние на число Нуссельта.